Location and Product Differentiation in a Duopoly with Externalities

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1. Introduction

The location decision by a firm depends on what other firms do. It is well known that locating far away in the geographic space, and locating far away in the characteristics' space of commodities (i.e. differentiating products) relaxes the toughness of competition. On the other side, by doing this, firms lose something in terms of positive spillovers they can benefit from surrounding competitors. In this paper I study how this trade-off is solved by rational firms, in a framework where product differentiation, cost externalities, and barriers to trade ultimately affect firms' location (both in the geographical and characteristics' space). To keep matters as simple as possible I consider a duopoly model.

If we make a thought experiment, and imagine a world without positive spillovers and localization economies, we should expect as a first approximation that firms spread out across space to meet final demand and maximize product differentiation. This is evidently at odds with the existence of industrial districts, that is clusters geographically concentrated producing very similar goods. So positive externalities should indeed exist. As Rosenthal - Strange (2004) put it, a distinction can be made among the scope and the sources of agglomeration economies.

As to the scope of local externalities, there seems to be statistically robust evidence that increasing the specialization of a given local area is beneficial in terms of productivity (see Cingano - Schivardi, 2004) as far as manu-

Received May 2004, Approved December 2004.

This paper draws on Chapter 3 of my doctoral thesis at the Department of Economics of the University of Pisa. I would like to thank for many valuable suggestions Carlo Casarosa, Stefano Federico, Elisa Giuliani, Luciano Greco, Luca Lambertini, Gianmarco Ottaviano, Paolo Scapparone, and seminar participants in Padova and Pisa. Comments from two anonymous referees *substantially improved the quality of the paper. All remaining errors are mine.*

facturing sectors are considered. In a simplified framework as the one analyzed in the present work, this can be analytically represented as a decrease in marginal cost as product differentiation shrinks. The less differentiated varieties are, the lower marginal cost is. The second kind of scope identified by the literature is geographical, that is externalities are spatially bounded. In the present paper this translates in the fact that the externality will be enjoyed only if firms are located in the same region.

As to the sources of localization economies, a vast literature dating back to Marshall has identified several channels: knowledge spillovers, input sharing, and labour market pooling are among them. Another source especially stressed in the business economics literature is the fact that peer pressure, and rivalry are more keenly felt when firms locate together (see, for example, Porter, 1998), and innovation could benefit from this. It seems fair to say that the intensity of all these elements, and their beneficial effects on productivity, should be inversely related to product differentiation and physical proximity.

The relation between product differentiation and costs, working through the operating of localization economies, is the first part of the story told in this paper. The second part deals with the way changes in barriers to trade (transport costs, tariffs, and, more generally, every per-unit-of-output cost afforded to export abroad) modify the equilibrium. A recurrent theme in the literature studying location of economic activities in space concerns the ambiguous effect of market integration. Some models suggest that we should expect first a concentration of the manufacturing sector as markets become more integrated and then, as a result of a further decrease of barriers to trade, dispersion again. Such a pattern can be found, for instance, in Krugman - Venables (1990), and Puga (1999). In the former model, the main ingredients are increasing returns to scale and imperfect competition, leading to a home market effect due to the asymmetry in market access in the two regions. Firms will more than proportionately locate in the thicker market, but this effect vanishes as transport costs go to zero. In the latter model, labour immobility acts as a dispersion force when transport costs are low enough, because it creates a wage differential between regions that induces at some point firms to locate where labour is cheaper.

In that class of models location is governed by pecuniary externalities. In this paper I show that a similar pattern of dispersion, agglomeration, and then dispersion again can be replicated in a framework where technological externalities are at work. In the present paper two firms, originally located separately, due to a decrease in transport costs may choose at some point to cluster together to exploit a technological externality which is stronger the less differentiated their output is. They do so in order to reduce costs of production. In our very stylized framework, when two firms locate in the same region and produce (even partially) homogeneous products we say they are forming an industrial district. Then, if transport costs keep decreasing, and if the intensity of the cost reduction coming from the externality is not high, at some point the competitive advantage deriving from the district (i.e. the fact of being located together, and produce similar commodities) vanishes. In that case firms are induced to maximize product differentiation and locate far from each other. In other words delocalization could be the outcome of strong market integration. This result crucially depends on the strength of the cost reduction coming from agglomeration and low product differentiation. If the cost reduction is strong, firms will still be located together for low transport costs. Consequently there are two classes of clusters. Some of them survive in a world of declining transport costs because the incentives to form the district are very strong. Others disappear. This distinction seems to match the fact that in Italian «superdistricts»[1] manufacturing employment decreased in the period 1991-2001 less than in the other districts (see on this point Signorini, 2004).

I use the differentiated duopoly model of Dixit (1979), and in particular the two regions' version developed in Belleflamme - Picard - Thisse (2000). In their paper, positing as we do a technological externality, there is the prediction that agglomeration is more likely as transport costs decrease. I show that this conclusion does not necessarily hold if localization economies depend on product differentiation: integration does not always lead to more spatial concentration. A second result concerns the relation between equilibrium product differentiation and the intensity of cost reduction. The higher the gain from agglomeration (and imitation), the higher the incentives to exploit the externality in equilibrium, and so the lower the degree of differentiation. The exact shape of optimum product differentiation varies with the form taken by the spillover function, which we assume to be either linear or quadratic. The common point underlying the two specifications is that, below a threshold of gains from agglomeration, optimum differentiation is maximal, so that firms do not find convenient to exploit localization economies. Above that threshold differentiation is non-increasing in costs, and firms locate in the same region.

We set up a game such that, in the first stage of the game, product differentiation is chosen. In the second stage firms simultaneously locate. Finally they compete in quantities. A conventional argument made to justify this sequence of actions is that quantities can be adjusted faster than product's characteristics or location. The degree of product differentiation is analytically represented in this paper through the parameter in the demand function linking own price to rival's quantity. This measure is closely related to cross elasticity of substitution.

Our modelling of product differentiation differs with respect to address

[1] Cannari - Signorini (2000) divide the total of Italian industrial districts in two subsets: «superdistricts», and the rest of districts. If one builds a series of indicators related to the traits usually characterizing industrial districts, superdistricts show a higher level for these indicators.

models (e.g. Hotelling) in that our paper retains only the strategic aspect of product differentiation (the incentive to differentiate in order to relax price competition), while it lacks the demand effect (the incentive to locate at the market centre in order to capture as many customers as possible) given the non-spatial nature of preferences. So firms would target maximum differentiation in the absence of any other incentive to lessen it. As suggested by Harrington (1995), the parameter of product differentiation that we employ, more than being linked to a measure of distance in the characteristics' space, is the counterpart of transport costs in Hotelling: the higher transport costs, the poorer substitutes the varieties are, because the higher is the loss incurred by customers in buying a product different from the preferred one. To sum up, when product differentiation changes in our model, we are actually changing the loss in utility coming from the consumption of a variety different from the preferred one, having the two firms symmetrically located somewhere in the characteristics' space (e.g. on the Hotelling line).

The idea that the product differentiation parameter in the Dixit (1979) duopoly model can be thought of as an empty box, to be filled with firms' choices, appears already in Lambertini - Rossini (1998).

The two aspects of location and product differentiation have been addressed together in Schmitt (1995). He considers two countries trading differentiated products subject to a barrier to trade. Preferences in each country are modelled à la Hotelling, as a segment of unit length, and each firm sets its location along the segment (fixing product differentiation), and in one of the two regions. The Nash equilibrium is imitation or maximum differentiation. Imitation is the outcome when the barrier to trade is sufficiently high and products are good substitutes (as captured by low transport costs on the Hotelling segment), and corresponds to a no trade pattern between the two regions. On the contrary, when the degree of substitution is relatively low and the barrier to trade is not high, firms maximally differentiate their products in equilibrium, and trade is established.

In the presence of spillover, the idea that marginal cost reduction is a decreasing function of product differentiation has already been investigated from a theoretical standpoint in Duranton (2000). Product differentiation is again à la Hotelling, and the magnitude of cost reduction depends on R&D investment.

The rest of the paper is organized as follows. Section 2 presents the basic structure of the model. In Section 3 the cost externality is explicitly modelled as a function of product differentiation, so that different equilibria are derived for various levels of transport costs. Section 4 concludes.

2. The model

We consider a differentiated duopoly model in the same spirit of Dixit (1979) and Belleflamme - Picard - Thisse (2000). The inverse demand function for the variety produced by firm i, without loss of generality, can be expressed as

$$p_i = \xi - q_i - \omega q_j \tag{1}$$

where j is the variety produced by the other firm and the variable $\omega \in [0, 1]$ represents product differentiation. When $\omega = 0$ varieties belong indeed to independent sectors, so that each firm is a monopolist, while when $\omega = 1$ varieties are the same homogeneous good. As in Lambertini - Rossini (1998), firms' strategies affect the degree of product differentiation ω. Contrary to them, firms differentiate the commodities they produce for free.

The economy is made of two regions, A and B. There are no fixed costs, and producing one unit of output requires c_K units of the numeraire if firms are located together, and c_S units if they are located separately. The cost reduction due to agglomeration is then $c_S - c_K$. The two regional markets are segmented, since firms set a specific quantity (competition takes place à la Cournot) in each region where the product is sold, and a cost equal to t is spent on each unit exported abroad. The most straightforward interpretation of t is in terms of transport costs, but every per-unit-of-output barrier to trade (such as a tariff or even some information costs) serves our purposes.

We set up a three stages game. The sequence of actions is as follows. In the first stage the production differentiation parameter ω is set. In the second stage firms choose simultaneously location, so that profits should be distinguished whether agglomeration (K) or dispersion (S) occurs. In the last (market) stage firms non-cooperatively compete in quantities. The equilibrium derived does not depend on the timing of firms' actions. In particular one may wonder whether sequential entry by firms may alter the solution. If we maintain the assumption that output in the third stage is chosen à la Cournot, then choosing product differentiation and/or location sequentially yields the same result of choosing them simultaneously. As it is shown below, payoffs (profits) depend in a symmetric way on product differentiation and location: even if a firm has a first mover advantage, maximization of its own profits with respect to these two variables also entails maximization of the opponent's.

2.1. *The market stage*

Let us consider firms i and j. They can be located either in A or in B. If the two firms are located, say, in region A, firm j's agglomeration profits $\Pi_{jK}(c_K, \omega, t)$ are

$$\begin{aligned} &\Pi_{jK}(c_K, \omega, t) = \Pi_{jA}(c_K, \omega, t) = \\ &(\xi - q_{jA} - \omega q_{iA} - c_K)q_{jA} + (\xi - q_{jB} - \omega q_{iB} - c_K - t)q_{jB} \end{aligned} \tag{2}$$

where c_K is marginal cost under agglomeration, and q_{iA}, q_{iB}, q_{jA}, q_{jB} are quantities sold by i and j in regions A and B respectively. If firms are located in different regions, with j being in B, its dispersion profits $\Pi_{jS}(c_S, \omega, t)$ will be

$$(3) \qquad \Pi_{jS}(c_S, \omega, t) = \Pi_{jB}(c_S, \omega, t) = \\ (\xi - q_{jB} - \omega q_{iB} - c_S)q_{jB} + (\xi - q_{jA} - \omega q_{iA} - c_S - t)q_{jA}$$

where c_S is marginal cost under dispersion. Expressions similar to (2) and (3) can be derived for firm i. Equilibrium variables q^*_{iA}, q^*_{iB}, q^*_{jA}, q^*_{jB} are calculated solving the four equations' system made of reaction functions, under agglomeration and dispersion respectively. Substituting in the inverse demand functions we get equilibrium prices[2]. Finally it is possible to derive equilibrium profits.

2.2. *The location stage*

Due to the perfect symmetry of the setting, payoffs (profits) accruing to both firms are identical in each spatial configuration: $\Pi_{iK}(c_K, \omega, t) = \Pi_{jK}(c_K, \omega, t)$ and $\Pi_{iS}(c_S, \omega, t) = \Pi_{jS}(c_S, \omega, t)$, so we can drop firms' subscripts. Equilibrium profits under agglomeration are:

$$(4) \qquad \Pi_K(c_K, \omega, t) = \frac{2(\xi - c_K)^2 - 2t(\xi - c_K) + t^2}{(2+\omega)^2}$$

Under dispersion, firms will get:

$$(5) \qquad \Pi_S(c_S, \omega, t) = \frac{2(\xi - c_S)^2(2-\omega)^2 + (4+\omega^2)t^2 - 2(\xi - c_S)(2-\omega)^2 t}{(2-\omega)^2(2+\omega)^2}$$

The equilibrium of the location stage will be the following: if $\Pi_K(c_K, \omega, t) > \Pi_S(c_S, \omega, t)$, agglomeration is the equilibrium; if the reverse inequality holds there is dispersion.

[2] One can easily check that the condition ensuring the non-negativity of quantities sold abroad (and the relative mark-ups of price over cost) is $t < \xi - c_K$ under agglomeration and $t < [(2-\omega)(\xi - c_S)]/2$ under dispersion. If $c_K < c_S$, the trade condition that allows export under dispersion is more restrictive than the trade condition under agglomeration. In both cases transport costs should be low enough for trade to occur.

2.3. *Benchmark solution*

Firms' profits are affected in the same manner by ω, because they are equal in a given spatial configuration. We are then allowed to say that the profit maximizing ω will always be achieved in equilibrium. Each firm finely tunes the design of its variety assuming the other does not do so (remember that the choice of product differentiation is simultaneous). Since product differentiation is not a costly activity, this process keeps going on until the profit maximizing ω is reached.

In Minerva (2003) it is proved that, without externalities ($c_K = c_S = c$), maximization of profits requires a degree of differentiation $\omega^* = 0$, and it is shown that when ω is strictly greater than zero dispersion profits are always higher than agglomeration ones. When $\omega = 0$, $\Pi_K(c, 0, t) = \Pi_S(c, 0, t)$, and this introduces an indeterminacy of location at this level of product differentiation. This is a consequence of the fact that for $\omega = 0$ the two varieties are indeed independent products, so that the two firms are monopolists:

$$\Pi_K(c, 0, t) = \Pi_S(c, 0, t) = \Pi^M$$

where Π^M are monopoly profits. Being located together or separately does not affect profits. However, if firms know they imperfectly maximize differentiation, due to a tremble (i.e. ω can be set almost equal to zero, but actually will be slightly greater than that) this is enough to make agglomeration profits always less than dispersion ones, and locating separately will be the optimal choice for $\omega = 0$ as well. This rules out the indeterminacy.

Assumption. When the equilibrium level of product differentiation is $\omega^* = 0$*, firms will prefer to locate separately due to a tremble.*

3. The cost externality

In this section the hypothesis is made that marginal cost under agglomeration is a function of product differentiation, $c_K(\omega)$, and decreases as product differentiation diminishes, with $0 \leq c_K(\omega) \leq c$, $c_K(0) = c$, $c_K(1) = 0$, and $c'_K(\cdot) < 0$. Marginal cost under dispersion c_S is constant and set equal to c. Because $c_K(1) = 0$, c is a measure of the maximum cost reduction that can be attained under agglomeration if the varieties produced are indeed a homogenous product. In other terms it represents the incentives to locate together and decrease product differentiation. In the Introduction, I discussed at length the rationale for introducing this type of externality, especially in relation with the formation of geographically concentrated clusters of firms producing similar products (industrial districts). Our aim is to assess when firms will cluster in the same region, and the degree of product differentiation maximizing profits.

Product differentiation has now two contrasting effects on profits in agglomeration. One is negative, since by increasing it the reduction in marginal cost is smaller. The other is positive, since an increase in product differentiation makes competition in the product market less intense, for a given level of marginal costs[3].

Given the structure of the game, there exists a continuum of possible final outcomes in the location and product differentiation space, $\Theta = \{[0, 1] \times \{K, S\}\}$, because the location possibilities are two (K or S), while the product differentiation variable ω can take any value in [0, 1]. In equilibrium the two firms cluster together whenever profits from agglomeration are strictly higher than those from dispersion, that is whenever

$$(6) \qquad \Pi_K(c_K(\omega^*), \omega^*, t) - \Pi_S(c, \omega^*, t)$$

is strictly positive, with ω^* being the equilibrium production differentiation level endogenously determined in the first stage of the game. ω^* is such that (6) is maximized. Since $\Pi_K(c_K(0), 0, t) = \Pi_S(c, 0, t) = \Pi^M$, maximization of (6) is indeed equivalent to

$$\underset{\omega}{\mathrm{Max}} \ \Pi_K(c_K(\omega), \omega, t)$$

There are two sorts of possible equilibria. The first occurs when $\omega^* > 0$ with the two firms being located together. Otherwise, if $\omega^* = 0$, firms locate separately because their profits amount to Π^M, and we go back to the benchmark solution. This means that in equilibrium *product similarity is always associated with agglomeration, while maximum product differentiation is associated with dispersion*. In the stylized framework of the paper, when $\omega^* > 0$ and the two firms are in the same region we have an industrial district. In what follows we determine ω^* for different specifications of the spillover function.

3.1. *Linear localization economies*

We prove the following relationship between c (the maximum gain due to agglomeration economies when $\omega = 1$) and optimum differentiation ω^* in the linear case, $c_K = (1 - \omega)c$.

Proposition 1. *Under Cournot competition and linear localization economies, the optimum level of product differentiation ω^* is the following.*

i) If $t < 4(\sqrt{5} - 2)\xi$, there exists a value c_L°,

[3] The profit function $\Pi_K(c_K, \omega, t)$ in (4) is decreasing in ω, keeping constant c_K and t.

$$c_L^{\circ} \equiv \xi - \frac{1}{2}t - \frac{1}{6}\sqrt{16\xi^2 - 16\xi t - t^2}$$

where $\xi/3 - t/6 < c_L^{\circ} < \xi/3$, *such that* $\omega^* = 1$ *for* $c > c_L^{\circ}$, *and* $\omega^* = 0$ *for* $c < c_L^{\circ}$.

ii) *If* $t > 4(\sqrt{5} - 2)\xi$, *then* $\omega^* = 0$.

Proof. See Appendix A.

The economic interpretation of this proposition suggests that, if the cost reduction attainable with agglomeration is significant, the product competition effect is overwhelmed by the cost saving one, and minimum differentiation is achieved. The reverse holds if c is low. Profit maximizing product differentiation in the linear case is either 0 or 1.

The threshold c_L° is a function of ξ and t. The parameter ξ represents the intensity of demand for the differentiated product. Let us focus on the derivative of c_L° with respect to transport costs. It is positive whenever

$$\frac{\partial c_L^{\circ}(\xi, t)}{\partial t} > 0 \Leftrightarrow t > 2(3\sqrt{2} - 4)\xi \cong 0.49\xi$$

and is negative in the opposite case. We plot the shape of the threshold in Fig. 1.

When $c < \xi/3$ and $t = 0$, firms produce independent products and separate in space, since it is better to maximize differentiation than to exploit the cost externality. The same holds when t is small. At some point, transport costs become high enough so that, in order to reduce the cost of production, firms exploit the technological externality locating together and setting $\omega = 1$. The switching to agglomeration is induced by the rising cost of exports. However, when t gets too big so that the foreign market cannot be easily accessed, firms maximize differentiation and locate separately to meet local demand. In Figure 2, we plot agglomeration profits if $c < \xi/3$ when ω is either 0 or 1. As long as $t_L < t < t_H$, profits with homogeneous products lay above profits with independent products, so that in equilibrium $\omega^* = 1$. When $t < t_L$ or $t > t_H$, the reverse is true and $\omega^* = 0$.

When $c > \xi/3$, the maximum gain that can be attained under agglomeration is so high that it is convenient to lessen product differentiation even when transport costs are null. The existence of the cluster is not threatened by declining transport costs: *those districts reducing a relevant part of their marginal cost under agglomeration still survive in an integrated economy*. In the preceding case, when $c < \xi/3$, low transport costs were a serious menace for the surviving of the cluster. The lower c the higher the threshold value of t below which the cluster vanished (see Fig. 1). Since the technological exter-

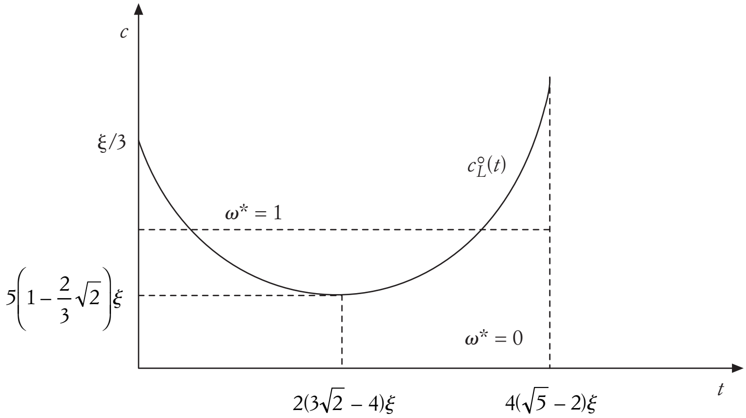

FIG. 1. The plot of the threshold $c_L^\circ(t)$ under linear localization economies.

nality was not strong in this case, once a component of costs was drastically reduced (namely the additional unit cost t afforded to export abroad) firms were induced to maximize again product differentiation[4].

The discussion of the linear case induces to distinguish among two types of clusters. Some of them are characterized by a high c, and survive even when t approaches zero. When c is small, but still greater than $5(1-(2/3)\sqrt{2})\xi$, we have the switching from agglomeration to dispersion as soon as $t < t_L$. Let us now consider an external authority wishing to predict the survival of the district at various levels of t. Assuming that the parameters ξ and t are known, this authority should in addition know c to compare it with the threshold c_L°. What happens if information about marginal cost is not known? In this case it is not possible to make predictions, and looking at the equilibrium level of product differentiation would not be helpful either. Provided agglomeration is the spatial equilibrium, the linear spillover assumption implies that firms always produce at $\omega^* = 1$ for every $c > c_L^\circ$. We now turn to a slightly different model, the quadratic spillover case, where something could be said looking at ω^*.

3.2. *Quadratic localization economies*

If we consider a quadratic specification for localization economies, $c_K = (1-\omega)^2 c$, the strength of the cost reduction under agglomeration is high-

[4] The observation that firms separate in space even when $\omega = 0$ depends on the assumption of a tremble. Removing it leads to indeterminacy of location, instead of dispersion, whenever the equilibrium level of ω is zero. Still in this case, agglomeration is less likely as markets become more integrated.

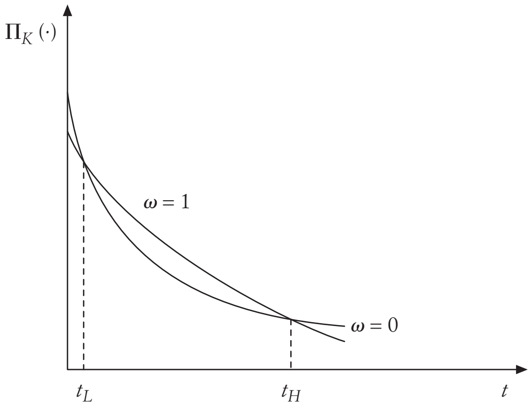

FIG. 2. Agglomeration profits $\Pi_K(c_K, \omega, t)$ under linear localization economies, with $5(1 - (2/3)\sqrt{2})\xi < c < \xi/3$, and $\omega = 0$ or $\omega = 1$.

er than in the linear case, for a given level of ω. The profit maximizing product differentiation obeys the following Proposition.

Proposition 2. *Under Cournot competition and quadratic localization economies, the optimum level of product differentiation* ω^* *is the following.*

i) *If* $t < 4(\sqrt{5} - 2)\xi$*, there exists a value* c_Q°*, with* $c_Q^\circ = (3/5)c_L^\circ$*, such that* $\omega^* = 0$ *for* $c \leq c_Q^\circ$*, and* $0 < \omega^*(c) < 2(\sqrt{2} < 1)$ *for* $c > c_Q^\circ$*. Moreover* $\omega^*(c)$ *is strictly increasing in* c*.*

ii) *If* $t > 4(\sqrt{5} - 2)\xi$*, then* $\omega^* = 0$.

Proof. See Appendix B.

We plot the shape of $c_Q^\circ(t)$ in Fig. 3.

The interpretation of this result is basically the same of the linear case. Under both circumstances $\omega^*(c)$ is non-decreasing in c. This feature of the model is robust to different analytical representations of localization economies, and the effect of the quadratic specification is to smooth out the relation between equilibrium product differentiation and c. Since the intensity of the cost reduction under agglomeration is stronger for a given level of product differentiation than in the linear case, the usual trade-off between cost savings and competition in the product market is solved at an equilibrium level of product differentiation always less than $2(\sqrt{2} - 1) < 1$. In addition $c_Q^\circ = (3/5)c_L^\circ$, so that the linear and quadratic thresholds have the same functional form with respect to t, apart from a multiplicative constant. Also in the quadratic specification delocalization could be the outcome of strong market integration.

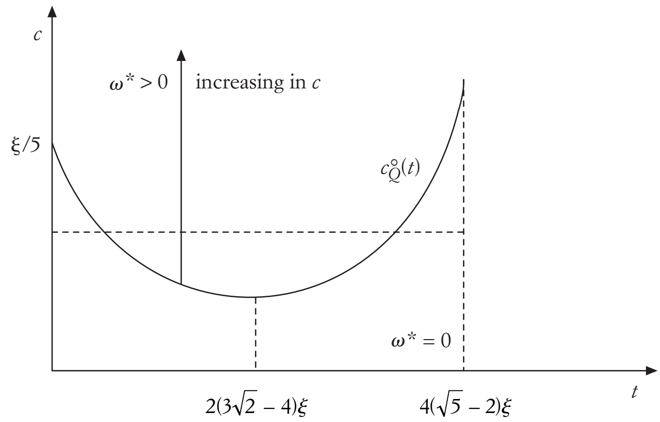

FIG. 3. The plot of the threshold $c_Q^\circ(t)$ under quadratic localization economies.

The main difference is that ω^* is a strictly increasing function of c. So the higher c the lower the level of equilibrium product differentiation associated to the cluster. As in the linear case, delocalization for a small t is less likely the higher c, because c_Q° is inversely related to t, if $t \in [0, 2(3\sqrt{2} - 4)\xi)$. This implies that, under the quadratic spillover function, the vanishing of the cluster is less likely the less differentiated varieties are in equilibrium, that is the more specialized the production of the two firms is. In contrast with the linear case, looking at equilibrium product differentiation could be of some help when assessing the likelihood of delocalization for a small t, even if the value of c is unknown. This is a distinctive feature of the quadratic spillover case.

The mechanism outlined above can be thought to match some evidence on the differential effects of declining trade barriers on the surviving of clusters of firms. Cannari - Signorini (2000) recognize that Italian districts are not all alike, and some of them are, in a sense, «superdistricts», provided they show the traits typical of districts to a greater extent. This can be fitted to the present paper by saying that superdistricts will produce a more specialized output, that is ω^* is higher. In the quadratic spillover case of this paper, the higher the incentives deriving from agglomeration (higher c) the higher ω^*. Moreover, lower trade barriers affect clusters less seriously the higher the specialization of output. Then our simple model proves that lower trade barriers affect less intensely superdistricts.

Some preliminary evidence (Signorini, 2004) supports the idea that the decrease in manufacturing employment over the period 1991-2001 was less intense in superdistricts than for the rest of Italian districts. Given that an important phenomenon during the period 1991-2001 has been increasing markets' integration at the international level (think, for instance, to increasing integration in the European Community), the theoretical results presented here might tell a story consistent with these stylized facts.

4. *Concluding remarks*

The idea that localization economies play an increasingly important role in determining location as transport costs fall was questioned in this paper. Our objective was to focus on the role played by endogenous product differentiation, and to study its impact assuming that localization economies do depend on the degree of product differentiation.

The usual positive effect of differentiation (relax competition) was mitigated by the fact that marginal cost under agglomeration was an increasing function of product differentiation itself. First we found that externalities lead to agglomeration and (at least) partial imitation when the gain is sufficiently high. This descends from the assumption that agglomeration economies are stronger the less differentiated varieties are. Equilibrium product differentiation was derived as a function of such a gain.

We then demonstrated that location depends on transport costs in a non-monotonic fashion. This result, which is in line with many models in the economic geography literature, is driven in this paper entirely by technological determinants. The robustness of the result to different specifications of the externality was successfully tested.

Our findings may be of some guidance when assessing the impact of markets' integration at the international or interregional level. We proved in a simple model how declining transport costs may lead industrial districts to lose their competitive advantage, if the strength of the marginal cost reduction under agglomeration is not high. This is interesting under two respects. In a first place, it neatly describes how transport costs interact with product differentiation and location choices. In a second place, we are able to distinguish among two types of clusters. The first, the superdistrict, survives even in a perfectly integrated economy since there are strong incentives to lessen product differentiation, whereas the second type declines.

Appendix A: Proof of Proposition 1

Step 1. We first prove that: if $c \geq \xi/3$, then $\omega^* = 1$; if $c \leq \xi/3 - t/6$, then $\omega^* = 0$. We decompose Π_K in the home component Π_K^h (profits made in the home market), and in the foreign component Π_K^f (profits made in the foreign market). Differentiating them separately we get the expression

$$(7) \quad \frac{\partial \Pi_K}{\partial \omega} = \frac{\partial \Pi_K^h}{\partial \omega} + \frac{\partial \Pi_K^f}{\partial \omega} = \frac{2[(3c - \xi)(\xi - c + \omega c) + (3c + t - \xi)(\xi - c - t + \omega c)]}{(2 + \omega)^3}$$

From an inspection of (7), if $c \geq \xi/3$ the derivative is positive for every ω. Rearranging the derivative of profits in the following form

$$\frac{\partial \Pi_K}{\partial \omega} = \frac{2\{[\xi - (1-\omega)c][(3c-\xi)+(3c+t-\xi)] - (3c+t-\xi)t\}}{(2+\omega)^3}$$

we see that if $c \leq (\xi - t)/3 - t/6$, the term $[(3c - \xi) + (3c + t - \xi)]$ is negative, and the derivative as well.

Step 2. Let us consider the case $\xi/3 - t/6 < c < \xi/3$. The point $\omega(c)$ solving the f.o.c. $\partial\Pi_K/\partial\omega = 0$ is unique. It is

$$\omega(c) = \frac{6c^2 + 4ct + t^2 - 8\xi c - 2\xi t + 2\xi^2}{c(6c + t - 2\xi)}$$

Substituting this value in $\partial^2\Pi_K/\partial\omega^2$, we obtain that the second order derivative is positive. Consequently it is a minimum, the candidates to maximize profits will be the extremes, $\omega = 0$ and $\omega = 1$. We evaluate profits in the extremes solving the inequality

$$\Pi_K(c_L^\circ, 0, t) \gtrless \Pi_K(c_L^\circ, 1, t) \tag{8}$$

The value of c_L° solving (8) is

$$c_L^\circ \gtrless \xi - \frac{1}{2}t - \frac{1}{6}\sqrt{16\xi^2 - 16\xi t - t^2} \tag{9}$$

To ensure existence, we have to assume that the term in the square root in (9) is positive, which requires $t < 4(\sqrt{5} - 2)\xi$.

Step 3. If $t > 4(\sqrt{5} - 2)\xi$, it can be easily checked that $\Pi_K(c, 0, t) > \Pi_K(0, 1, t)$.

Appendix B: Proof of Proposition 2

Proving that $\omega^* < 1$, simply requires to evaluate $\partial\Pi_K/\partial\omega$ at $\omega = 1$, and see that it is negative whatever is c.

Step 1. As in the proof of Proposition 1, we consider home and foreign components of profits separately: ω_h^* is the value of product differentiation maximizing the home component $\Pi_K^h(\xi, c, \omega)$, and ω_f^* maximizes the foreign component $\Pi_K^f(\xi, c, \omega, t)$ which is a function of transport costs as well.

Partially differentiating the home component,

$$\frac{\partial \Pi_K^h}{\partial \omega} = \frac{2[(1-2\omega+\omega^2)c-\xi][\xi-(5-4\omega-\omega^2)c]}{(2+\omega)^3} \equiv h(\xi, c, \omega)$$

When $c < \xi/(5-4\omega-\omega^2)$, this derivative is negative. When $c > \xi/(5-4\omega-\omega^2)$, this derivative is positive. Thus, as long as $c \leq \xi/5$, $\omega_h^* = 0$. For $c > \xi/5$, $\omega_h^* > 0$. Solving with respect to ω the f.o.c.

$$\frac{\partial \Pi_K^h}{\partial \omega} = 0$$

we get the explicit solution

$$\omega_h^*(\xi, c) = \frac{\sqrt{9c-\xi} - 2\sqrt{c}}{\sqrt{c}} \tag{10}$$

This point is a maximum because the second order partial derivative is negative at $\omega_h^*(\xi, c)$. In addition $\omega_h^*(\xi, c)$ is increasing in c and decreasing in ξ. We can bound from above optimal product differentiation for the home component of profits, since $c < \xi - t < \xi$. Actually, $\omega_h^*(\xi, \xi) = 2(\sqrt{2} - 1) \simeq 0.83$.

Then note that $\Pi_K^f(\xi, c, \omega, t) = \Pi_K^h(\xi', c, \omega)$, where $\xi' \equiv \xi - t$. So all the results derived above apply also for the foreign component. In particular $\omega_h^*(\xi, c) < \omega_f^*(\xi, c) \equiv \omega_h^*(\xi', c)$. The upper bound is still $2(\sqrt{2} - 1)$.

Step 2. When $(\xi - t)/5 < c < \xi/5$, foreign profits evaluated at $\omega = 0$ increase in ω, i.e. $h(\xi', c, 0) > 0$, while home profits do not, i.e. $h(\xi, c, 0) < 0$. Decreasing differentiation has a beneficial effects in terms of the foreign market only. Since $\partial^2 \Pi_K^h(\xi, c, \omega)/\partial\omega\partial c \equiv \partial h(\xi, c, \omega)/\partial c > 0$, when $c < \xi/5$, by continuity there exists a threshold value of c, which we call c_Q°, making zero $h(\xi, c, 0) + h(\xi', c, 0)$ the derivative of total profits at $\omega = 0$. For $c \geq c_Q^\circ$ it must be

$$h(\xi, c, 0) + h(\xi', c, 0) \geq 0 \tag{11}$$

Solving (11), and considering only the solution satisfying the constraint $c < \xi - t$, then $\omega^* = 0$ for

$$c \leq \frac{3}{5}\left(\xi - \frac{1}{2}t - \frac{1}{6}\sqrt{16\xi^2 - 16\xi t - t^2}\right) \equiv c_Q^\circ = \frac{3}{5}c_L^\circ$$

and $\omega^* > 0$ otherwise.

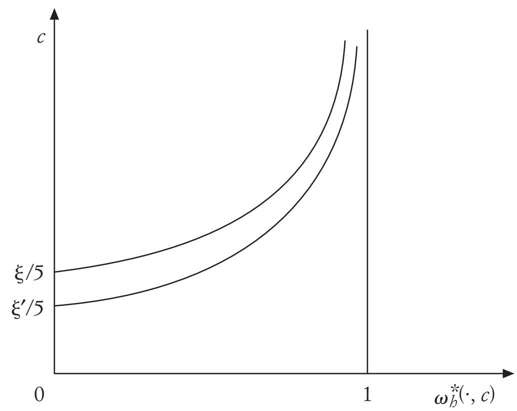

FIG. 4. The solutions $\omega_h^*(\xi, c)$ and $\omega_h^*(\xi', c)$.

Step 3. $\omega^*(c)$ maximizing total profits solves the equation

$$h(\xi, c, \omega) + h(\xi', c, \omega) = 0$$

for every $c \geq c_Q^\circ$.

It is possible to argue that

$$\omega_h^*(\xi, c) < \omega^*(c) < \omega_h^*(\xi', c) \equiv \omega_f^*(\xi, c) \tag{12}$$

because $\omega_h^*(\xi, c)$ is decreasing in ξ, so that $\omega^*(c)$ will lie between the two curves in Fig. 4. It follows immediately that $\omega^*(c) < 2(\sqrt{2} - 1)$.

We now impose the condition

$$\frac{\partial^2 \Pi_K(\xi, c, \omega, t)}{\partial\omega\partial c} = \frac{\partial h(\xi, c, \omega)}{\partial c} + \frac{\partial h(\xi', c, \omega)}{\partial c} > 0$$

which is true as long as

$$c < \frac{3(\xi + \xi')}{10 - 18\omega + 6\omega^2 + 2\omega^3} \tag{13}$$

Making explicit c in (10), we get

$$c = \frac{\xi}{9 - (\omega + 2)^2}$$

and

$$\frac{3(\xi+\xi')}{10-18\omega+6\omega^2+2\omega^3} > \frac{\xi}{9-(\omega+2)^2}$$

Since $\omega_b^*(\xi, c) > \omega^*(\xi, c)$ then $\omega^*(\xi, c)$ checks (13), and it is strictly increasing in c.

Step 4. When $t > 4(\sqrt{5}-2)\xi$, (11) is negative, so that $\omega^* = 0$.

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Summary: Location and Product Differentiation in a Duopoly with Externalities (J.E.L. D43, F12, L13)

In a duopoly model, marginal cost decreases if product differentiation does so, provided that firms are located in the same region. In equilibrium, the larger the cost that can be saved under agglomeration, the less differentiated varieties are. A pattern of dispersion, agglomeration, and then dispersion again can emerge as transport costs go down due to the interplay between product differentiation and the cost externality.